

Baruun dala a 11-a

①.

$$\text{I 1. a) } 2 \log_3 \sqrt{77} - \frac{1}{3} \log_3 \frac{7^3 \cdot 3^6}{125} + 2 \log_3 \frac{3}{\sqrt{55}} =$$
$$= \log_3 \sqrt{77}^2 - \log_3 \sqrt[3]{\frac{7^3 \cdot 3^6}{5^3}} + \log_3 \left(\frac{3}{\sqrt{55}} \right)^2 = (2p)$$

$$= \log_3 77 - \log_3 \frac{7 \cdot 3^2}{5} + \log_3 \frac{9}{\sqrt{55}} = (2p)$$

$$= \log_3 \left(\frac{7}{7} \cdot \frac{8}{7 \cdot 3^2} \cdot \frac{9}{\sqrt{55}} \right) = \log_3 1 = 0 \quad (1p)$$

$$\text{b) } \log_2 3 \cdot \log_3 16 = \frac{\log_2 16}{\log_2 3} = 4 \quad (2p)$$

$$\text{c) } 3^{2 - \log_3 10} - 4^{1 - \log_4 5} + 6^{\log_6 9 - 1} =$$
$$= \frac{3^2}{3^{\log_3 10}} - \frac{4^1}{4^{\log_4 5}} + \frac{6^{\log_6 9}}{6^1} = (2p)$$
$$= \frac{3^2}{10} - \frac{4}{5} + \frac{9}{6} = \frac{27 - 24 + 45}{30} = \frac{48}{30} = \frac{16}{10} = \frac{8}{5} \quad (1p)$$

(2p).

$$2. a) \sqrt{3-x} - \sqrt{10-x} = -1$$

$$c \in \begin{cases} 3-x \geq 0 \\ 10-x \geq 0 \end{cases} \Rightarrow x \in (-\infty, 3]. \quad (1p)$$

$$\sqrt{3-x} + 1 = \sqrt{10-x} \quad \text{ridicăm la pătrat} \quad (1p)$$

$$3 - \cancel{x} + 2\sqrt{3-x} + 1 = 10 - \cancel{x} \quad (1p)$$

$$2\sqrt{3-x} + 4 = 10$$

$$2\sqrt{3-x} = 6 \quad | :2$$

$$\sqrt{3-x} = 3 \quad (1p)$$

$$3-x = 9$$

$$x = -6 \in (-\infty, 3] \quad (1p)$$

$$b) \frac{2^x}{2^3} + \frac{2^y}{2^2} + \frac{2^x}{2^1} = 448. \quad \text{notăm } 2^x = t > 0 \quad (2p)$$

$$\frac{t}{8} + \frac{t}{4} + \frac{t}{2} = 448 \quad | \cdot 8$$

$$t + 2t + 4t = 448 \cdot 8$$

$$7t = 448 \cdot 8 \quad | :7$$

$$t = 64 \cdot 8$$

$$2^x = 2^9 \quad (2p)$$

$$x = 9 \quad (1p)$$

$$c) \quad C \in : x > 0 \quad \left| \quad \begin{array}{l} \lg x \neq -1 \\ \lg x \neq -3 \end{array} \right. \Rightarrow x \in (0, \infty) - \left\{ \frac{1}{10}, \frac{1}{10^3} \right\} \quad (1P)$$

notes $\lg x = t$

$$\frac{1}{1+t} + \frac{3}{3+t} = 2$$

$$3+t+3(1+t) = 2(1+t)(3+t)$$

$$3+t+3+3t = 2(3+t+3t+t^2)$$

$$4t+6 = 6+8t+2t^2 \quad (1P)$$

$$2t^2+4t=0$$

$$2t(t+2)=0 \quad \begin{cases} t=0 \\ t=-2 \end{cases} \quad (1P)$$

$$\lg x = 0 \Rightarrow x = 1 \in (0, \infty) - \left\{ \frac{1}{10}, \frac{1}{10^3} \right\} \quad (1P)$$

$$\lg x = -2 \Rightarrow x = \frac{1}{10^2} \in (0, \infty) - \left\{ \frac{1}{10}, \frac{1}{10^3} \right\} \quad (1P)$$

$$3 a) \quad p = \frac{n \text{ cas favorabile}}{n \text{ cas posibil}} = \frac{36}{81} = \frac{4}{9} \quad (2P)$$

$$n \text{ cas posibil} = 81 \quad (1P)$$

$$n \text{ cas favorabile} = 36 \quad (1P)$$

$$\overline{ab}, a \in \Delta_6 = \{1, 2, 3, 6\}$$

$$\overline{1b}, \overline{2b}, \overline{3b}, \overline{6b} - 9 \cdot 4 = 36 \text{ numere}$$

$$b) \quad \left(\sqrt[3]{3} + 2\sqrt[5]{2} \right)^{96}$$

$$T_{k+1} = C_n^k a^{n-k} b^k = C_{96}^k (\sqrt[3]{3})^{96-k} (2\sqrt[5]{2})^k \quad (1p)$$

$$T_{k+1} \in \mathbb{Q} \Rightarrow \begin{matrix} 96-k \in M_3 \\ k \in M_5 \end{matrix} \quad (1p)$$

$$k \in M_5 = \{0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95\} \quad (1p)$$

$$96-k \in M_3 \Rightarrow k \in \{0, 15, 30, 45, 60, 75, 90\} \quad (1p)$$

dezvoltarea are 7 termeni rationali. (1p)

$$c) \quad C_n^k = \frac{n!}{k!(n-k)!} \quad A_n^k = \frac{n!}{(n-k)!} \quad P_k = k!$$

$$C_6^3 = \frac{6!}{3!3!} = \frac{4 \cdot 5 \cdot 6}{2!} = 20 \quad (1p)$$

$$A_4^2 = \frac{4!}{2!} = 3 \cdot 4 = 12 \quad (1p)$$

$$P_4 = 4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24 \quad (1p)$$

$$C_6^3 + A_4^2 + P_4 = 20 + 12 + 24 = 56 \quad (2p)$$

II 4.

$$\left(3x^2 + \frac{1}{\sqrt[4]{x^2}}\right)^{15}$$

$$T_{h+1} = C_{15}^h (3x^2)^{15-h} \cdot \left(\frac{1}{\sqrt[4]{x^2}}\right)^h = (3p)$$

$$= C_{15}^h \cdot 3^{15-h} \cdot (x^2)^{15-h} \cdot \left(x^{-\frac{2}{4}}\right)^h = (3p)$$

$$= C_{15}^h \cdot 3^{15-h} \cdot x^{30-2h-\frac{h}{2}} \quad (3p)$$

$$x^{30-2h-\frac{h}{2}} = x^0 \Rightarrow 30-2h-\frac{h}{2}=0 \quad (3p)$$

$$\Rightarrow 60-4h-h=0 \Rightarrow 60=5h \Rightarrow h=12 \Rightarrow T_{13} \quad (2p) \quad (1p)$$

2. a) AB:

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} \quad (2p)$$

$$\frac{x+2}{-1+2} = \frac{y+3}{y+3}$$

$$\frac{x+2}{1} = \frac{y+3}{10}$$

$$10x+20=y+3 \quad (2p)$$

$$y=10x+17 \text{ ec } AB \Rightarrow w_{AB}=10 \quad (1p)$$

b) fie d₁ dreapta c ut at  d₁||AB $\Rightarrow$

$$\Rightarrow w_{d_1}=w_{AB}=10 \quad (2p)$$

ec $d_1: y - y_0 = m(x - x_0) \quad C(x_0, y_0) \quad (2p)$
 $m = 10$

$$y - 1 = 10(x + 4)$$

$$y - 1 = 10x + 40$$

$$y = 10x + 41 \quad (1p)$$

c) fie d_2 înălțimea din C
 $d_2 \perp AB \Rightarrow m_{d_2} \cdot m_{AB} = -1 \Rightarrow m_{d_2} = -\frac{1}{10} \quad (2p)$

$$y - y_0 = m(x - x_0) \quad C(x_0, y_0) \quad (2p)$$

$$m = -\frac{1}{10}$$

$$y - 1 = -\frac{1}{10}(x + 4)$$

$$y - 1 = -\frac{x}{10} - \frac{4}{10}$$

$$y = -\frac{x}{10} + \frac{6}{10} \quad (1p)$$

3. Fie AB de ecuație: $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \quad (3p)$

$$\frac{x - 1}{8 - 1} = \frac{y - 2}{8 - 2} \quad (3p)$$

$$\frac{x - 1}{8} = \frac{y - 2}{6} \quad | \cdot 2 \Rightarrow \frac{x - 1}{4} = \frac{y - 2}{3} \Rightarrow (3p)$$

$$\Rightarrow 3x - 3 = 4y - 8 \Rightarrow 3x - 4y + 5 = 0 \text{ ec } (AB) \quad (1p)$$

$$C \in AB \Leftrightarrow 3 \cdot 5 - 4 \cdot 5 + 5 = 0 \Leftrightarrow 15 - 20 + 5 = 0$$

Adică $\Rightarrow A, B, C$ coliniare. $(5p)$