

Barem cloze a10-a

(1)

$$I. a_n = a_1 + (n-1)h, n \geq 2$$

$$S_n = \frac{(a_1 + a_n)n}{2}$$

$$a_2 = a_1 + h$$

$$a_5 = a_1 + 4h$$

$$2a_1 + 5h = 14$$

(3p)

$$S_{15} = \frac{(a_1 + a_{15})15}{2}$$

$$S_{15} = \frac{(a_1 + a_1 + 14h)15}{2} \quad (3p)$$

$$S_{15} = (a_1 + 7h)15$$

$$(1p) (a_1 + 7h) \cdot 15 = 240 \Rightarrow$$

$$a_1 + 7h = 16$$

$$(3p) \begin{cases} a_1 + 7h = 16 \\ 2a_1 + 5h = 14 \end{cases} \cdot 2 \Leftrightarrow \begin{cases} 2a_1 + 14h = 32 \\ 2a_1 + 5h = 14 \end{cases} \quad (3p)$$

$$\underline{9h = 18} \Rightarrow h = 2$$

$$\begin{matrix} a_1 + 7h = 16 \\ h = 2 \end{matrix} \Rightarrow a_1 + 14 = 16 \Rightarrow a_1 = 2 \quad (2p)$$

$$2. (f \circ g)(x) = f(g(x)) = f(x-3) = (x-3)^2 + f(x-3) - 7 = x^2 - 6x + 9 + f(x-3) - 7 = x^2 + x - 19 \quad (5p)$$

$$\begin{matrix} x^2 + x - 19 = -7 \\ x^2 + x - 12 = 0 \end{matrix} \quad (5p)$$

$$\Delta = 49$$

$$x_{1,2} = \frac{-1 \pm 7}{2}$$

$$\begin{matrix} -4 \\ 3 \end{matrix}$$

$$x \in \{-4, 3\}$$

(5p)

$$3. a) \quad \Delta = 81 - 8 = 73$$

$$-\frac{\Delta}{4a} = -\frac{73}{8} \quad (1p)$$

$$a > 0 \quad \text{Im} f = \left[-\frac{\Delta}{4a}, \infty \right) \quad (2p)$$

$$\text{Im} f = \left[-\frac{73}{8}, \infty \right) \quad (2p)$$

$$b) \quad V(x_v, y_v)$$

$$x_v = -\frac{b}{2a} = -\frac{9}{4} \quad (3p)$$

$$y_v = -\frac{\Delta}{4a} = -\frac{73}{8}$$

$$V\left(-\frac{9}{4}, -\frac{73}{8}\right) \quad (2p)$$

$$c) \quad \frac{f(x)}{8x+4} \leq 1$$

$$\frac{2x^2 + 9x + 1}{8x + 4} - 1 \leq 0$$

$$\frac{2x^2 + 9x + 1 - 8x - 4}{8x + 4} \leq 0$$

$$\frac{2x^2 + x - 3}{8x + 4} \leq 0. \quad (1p)$$

$$2x^2 + x - 3 = 0$$

$$\Delta = 25$$

$$x_{1/2} = \frac{-1 \pm 5}{4} \begin{cases} -\frac{6}{4} = -\frac{3}{2} \\ \frac{4}{4} = 1 \end{cases} \quad (1p)$$

$$8x + 4 = 0$$

$$x = -\frac{1}{2} \quad (1p)$$

x	$-\infty$	$-\frac{3}{2}$	$-\frac{1}{2}$	1	∞
$2x^2+x-3$	+	+	0	—	+
$8x+4$	—	—	—	0	+
$\frac{2x^2+x-3}{8x+4}$	—	—	0	+	+

$$x \in (-\infty, -\frac{3}{2}] \cup (-\frac{1}{2}, 1] \quad (2p)$$

$$\text{II } 1. \quad x \in (\pi, \frac{3\pi}{2}) \Rightarrow \begin{cases} \sin x < 0 \\ \cos x < 0 \end{cases} \quad (3p)$$

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{16}{25} + \cos^2 x = 1 \Rightarrow \cos^2 x = \frac{9}{25} \quad \left| \begin{array}{l} \cos x < 0 \end{array} \right. \Rightarrow \cos x = -\frac{3}{5} \quad (2p)$$

$$\sin 2x = 2 \sin x \cos x = 2 \cdot \left(-\frac{4}{5}\right) \cdot \left(-\frac{3}{5}\right) = \frac{24}{25} \quad (5p)$$

$$\cos 2x = \cos^2 x - \sin^2 x = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25} \quad (5p)$$

$$2. \quad \begin{array}{l} \vec{u}, \vec{v} \text{ coliniari} \\ \vec{u} = x_1 \vec{i} + y_1 \vec{j} \\ \vec{v} = x_2 \vec{i} + y_2 \vec{j} \end{array} \quad \left| \Rightarrow \frac{x_1}{x_2} = \frac{y_1}{y_2} \Rightarrow \right. \quad (5p)$$

$$\frac{a}{8} = \frac{1}{2} \Rightarrow 2a = 8 \Rightarrow a = 4. \quad (5p)$$

$$3. \quad \frac{a}{\sin A} = 2R \quad \frac{BC}{\sin A} = 2R \quad (5p)$$

$$\begin{array}{l} \sin^2 A + \cos^2 A = 1. \\ \sin A > 0 \end{array} \quad \left| \Rightarrow \sin^2 A = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow \right.$$

$$\sin A = \frac{\sqrt{3}}{2}. \quad (5p)$$

$$\frac{12}{\frac{\sqrt{3}}{2}} = 2R \Rightarrow \frac{24}{\sqrt{3}} = 2R \Rightarrow R = \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}. \quad (5p)$$