

Colegiul National ION NECULCE

Test de transfer clasa a 10-a februarie 2026

Subiectul I (40p)

1. Stiind ca $\log_3 2 = a$, aratati ca $\log_{16} 24 = \frac{1+3a}{4a}$.
2. Sa se determine $\operatorname{Re} z$, $\operatorname{Im} z$ si $|z|$ daca :
 - a) $z = \left(\frac{1-2i}{1+i}\right)^2 + \left(\frac{1+2i}{1-i}\right)^2$
 - b) $z = i^7 + i^{18} + i^{35} + i^{97} + i^{100} + (-i)^7 + (-i)^{15} + (-i)^{35} + (-i)^{97}$.
3. Calculati : $3^{2-\log_3 10} - 4^{1-\log_4 5} + 6^{\log_6 9-1}$

Subiectul II (50p)

4. Rezolvati ecuatiile urmatoare :
 - a) $\sqrt{5x} - \sqrt{x-1} = \sqrt{3x+1}$
 - b) $\left(\frac{9}{4}\right)^{3x+1} \cdot \sqrt{\left(\frac{8}{27}\right)^{6x}} = \frac{3}{2}$
 - c) $36^x + 7 \cdot 2^x \cdot 3^x - 78 = 0$
 - d) $\log_{0,(3)}(3x^2 - 19x + 27) = -3$
 - e) $\lg(x^2 - 6x + 5) - \lg(3 - x) = \lg 3$

Nota. Se acorda 10 puncte din oficiu. Timp de lucru 60 minute. Fiecare cerinta 10 puncte.

Subiectul I

$$1. \log_{10} 24 = \frac{\log_3 24}{\log_3 10} = \frac{\log_3 (2^3 \cdot 3)}{4 \log_3 2} = \frac{1 + 3a}{4a}.$$

$$\begin{aligned} 2. a) z &= \left(\frac{(1-2i)(1-i)}{1-i^2} \right)^2 + \left(\frac{(1+2i)(1+i)}{1-i^2} \right)^2 = \\ &= \left(\frac{1-i-2i-2}{2} \right)^2 + \left(\frac{1+i+2i-2}{2} \right)^2 = \frac{(-1-3i)^2}{4} + \frac{(-1+i)^2}{4} = \\ &= \frac{1+6i-9+1-6i-9}{4} = \frac{-16}{4} = -4 \end{aligned}$$

$$\operatorname{Re} z = -4$$

$$\operatorname{Im} z = 0$$

$$|z| = 4$$

$$b) z = \cancel{i^{18}} + \cancel{i^{18}} + \cancel{i^{35}} + \cancel{i^{94}} + \cancel{i^{100}} - \cancel{i^{15}} - \cancel{i^{38}} - \cancel{i^{94}} =$$

$$i^{16} \cdot i^2 + 1 - i^{12} \cdot i^3 =$$

$$= -1 + 1 - (-i) = i$$

$$\operatorname{Re} z = 0$$

$$\operatorname{Im} z = 1$$

$$|z| = 1$$

$$3. \frac{3^2}{3 \log_3 10} - \frac{4^1}{4 \log_4 5} + \frac{6 \log_6 9}{6^1} = \frac{3}{10} - \frac{6}{5} + \frac{5}{6} =$$

$$= \frac{27-24+45}{30} = \frac{48}{30} = \frac{8}{5}.$$

II 4 a) $5x \geq 0$
 $x-1 \geq 0$
 $3x+1 \geq 0$

$$\sqrt{5x} = \sqrt{x-1} + \sqrt{3x+1} \quad \uparrow^2$$

$$5x = x - 1 + 2\sqrt{x-1} \cdot \sqrt{3x+1} + 3x + 1$$

$$5x = 5x + 2\sqrt{x-1} \cdot \sqrt{3x+1}$$

$$0 = 2\sqrt{x-1} \cdot \sqrt{3x+1}$$

$x=1$ soluție
 $x=-\frac{1}{3}$ nu e soluție

b) $\left(\frac{3}{2}\right)^{6x+2} \cdot \left(\frac{2}{3}\right)^{\frac{18x}{2}} = \frac{3}{2}$

$$\left(\frac{3}{2}\right)^{6x+2} \cdot \left(\frac{3}{2}\right)^{-9x} = \frac{3}{2}$$

$$\left(\frac{3}{2}\right)^{-3x+2} = \frac{3}{2}$$

$$\Rightarrow -3x+2=1 \Rightarrow x=\frac{1}{3}$$

c) $6^x = t > 0$. $t^2 + 7t - 48 = 0$ $t_1 = 6$
 $\Delta = 49 + 384 = 361 = 19^2$ $t_2 = -13 < 0$

$$6^x = 6 \Rightarrow x=1$$

d) c.e: $3x^2 - 19x + 27 > 0$

$$\lg_{\frac{1}{3}}(3x^2 - 19x + 27) = \lg_{\frac{1}{3}}\left(\frac{1}{3}\right)^{-3}$$

$$3x^2 - 19x + 27 = 27 \Rightarrow 3x^2 - 19x = 0$$

$x=0$ soluție
 $x=\frac{19}{3}$

e) c.e: $\begin{cases} x^2 - 6x + 5 > 0 \\ 3-x > 0 \end{cases}$

$$\lg \frac{x^2 - 6x + 5}{3-x} = \lg 3$$

$$\frac{x^2 - 6x + 5}{3-x} = 3$$

$$x^2 - 6x + 5 = 9 - 3x$$

$$x^2 - 3x - 4 = 0$$

$x=-1$ soluție

$x=4$ nu e soluție

Subiectul I (40p)

1. Se considera matricea $A(a) = \begin{pmatrix} a+1 & 0 & 0 \\ 0 & 1 & \ln(a+1) \\ 0 & 0 & 1 \end{pmatrix}$, unde a este un parametru real pozitiv.
- Calculati $\det A(1)$
 - Demonstrati ca $A(a) \cdot A(b) = A(ab + a + b)$, pentru orice numere reale pozitive a si b .
 - Determinati numarul real pozitiv a pentru care $A(a) \cdot A(a) \cdot A(a) = A(7)$.

2. Sa se afle matricea X daca :

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 0 & 2 \\ 2 & 1 \end{pmatrix}$$

Subiectul II (50p)

3. Calculati limitele urmatoare :

a) $\lim_{x \rightarrow 0} \frac{\ln(1+x^3)}{x^4+x^3} =$

b) $\lim_{x \rightarrow 3} \frac{x^2-4x+3}{x^2-9} =$

c) $\lim_{x \rightarrow \infty} (x\sqrt{3} - \sqrt{3x^2 + 4x + 1}) =$

4. Determinati asimptotele functiilor urmatoare :

$$f: \mathbb{R} \setminus \{2\} \rightarrow \mathbb{R}, f(x) = \frac{\sqrt{x^2+3}}{x-2}$$

5. Sa se afle parametrii reali a si b pentru care functia urmatoare este continua in punctul specificat, anume $x = 0$.

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$f(x) = \begin{cases} \frac{\sin ax}{bx} & , x < 0 \\ \frac{e^{ax}-1}{6x} & , x > 0 \\ \frac{2}{3} & , x = 0 \end{cases}$$

Nota. Se acorda 10 puncte din oficiu. Timp de lucru 60 minute. Fiecare cerinta 10 puncte.

BAREM CLASA II - TRANSFER

Subiectul I (40p)

$$1) a) A(1) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & \ln 2 \\ 0 & 0 & 1 \end{pmatrix} \quad (2p)$$

$$\det A(1) = 2 + 0 + 0 - (0 + 0 + 0) = 2 \quad (8p)$$

$$b.) A(a) \cdot A(b) = \begin{pmatrix} a+1 & 0 & 0 \\ 0 & 1 & \ln(a+1) \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} b+1 & 0 & 0 \\ 0 & 1 & \ln(b+1) \\ 0 & 0 & 1 \end{pmatrix} = (3p)$$

$$= \begin{pmatrix} (a+1)(b+1) & 0 & 0 \\ 0 & 1 & \ln(a+1) + \ln(b+1) \\ 0 & 0 & 1 \end{pmatrix} = (4p)$$

$$= \begin{pmatrix} ab+a+b+1 & 0 & 0 \\ 0 & 1 & \ln(ab+a+b+1) \\ 0 & 0 & 1 \end{pmatrix} = (2p)$$

$$= A(ab+a+b) \quad (2p)$$

$$c.) A(a) \cdot A(a) \stackrel{b)}{=} A(a^2 + 2a) \quad (2p)$$

$$A(a) \cdot A(a) \cdot A(a) = A(a(a^2 + 2a) + a^2 + 2a + a) \quad (2p)$$

$$= A(a^3 + 3a^2 + 3a) \quad (1p)$$

$$A(a^3 + 3a^2 + 3a) = A(7) \Rightarrow a^3 + 3a^2 + 3a = 7 \quad (2p)$$

$$(a+1)^3 = 8 \Rightarrow a+1 = 2 \Rightarrow a = 1 \quad (3p)$$

$$2) \text{ Fi } X = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} \quad (2p)$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} = \begin{pmatrix} a-c & b-d \\ c+e & d+f \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 2 & 1 \end{pmatrix} \quad (3p)$$

$$\begin{cases} a-c=0 & \Rightarrow c=a \\ b-d=2 & \Rightarrow d=b-2 \\ c+e=2 & \Rightarrow e=2-a \\ d+f=1 & \Rightarrow f=3-b \end{cases} \quad (4p)$$

$$X = \begin{pmatrix} a & b \\ a & b-2 \\ 2-a & 3-b \end{pmatrix} \quad \forall a, b \in \mathbb{R} \quad (1p)$$

Subiectul II (50p)

$$3. a) \lim_{x \rightarrow 0} \frac{\ln(1+x^3)}{x^4+x^3} \stackrel{0}{=} \lim_{x \rightarrow 0} \underbrace{\frac{\ln(1+x^3)}{x^3}}_{\downarrow 1} \cdot \frac{x^3}{x^4+x^3} = (3p)$$

$$= \lim_{x \rightarrow 0} \frac{x^3}{x^3(x+1)} \stackrel{(2p)}{=} \lim_{x \rightarrow 0} \frac{1}{x+1} = 1 \quad (3p)$$

$$b) \lim_{x \rightarrow 3} \frac{x^2-4x+3}{x^2-9} \stackrel{0}{=} \lim_{x \rightarrow 3} \frac{(x-1)(x-3)}{(x-3)(x+3)} = (5p)$$

$$= \lim_{x \rightarrow 3} \frac{x-1}{x+3} \stackrel{(2p)}{=} \frac{2}{6} = \frac{1}{3} \quad (1p)$$

$$c) \lim_{x \rightarrow \infty} (x\sqrt{3} - \sqrt{3x^2+4x+1}) \stackrel{\infty-\infty}{=} \quad (2p)$$

$$= \lim_{x \rightarrow \infty} \frac{3x^2 - (3x^2+4x+1)}{x\sqrt{3} + \sqrt{3x^2+4x+1}} = \quad (2p)$$

$$= \lim_{x \rightarrow \infty} \frac{-4x-1}{x\sqrt{3} + \sqrt{3x^2+4x+1}} = \quad (2p)$$

$$= \lim_{x \rightarrow \infty} \frac{x(-4 - \frac{1}{x})}{x(\sqrt{3} + \sqrt{3 + \frac{4}{x} + \frac{1}{x^2}})} \stackrel{(2p)}{=} \frac{-4\sqrt{3} - 2\sqrt{3}}{2 \cdot 3} = -\frac{2\sqrt{3}}{3} \quad (2p) \quad 2/3$$

$\lim_{x \rightarrow \infty} f(x) \stackrel{(2p)}{=} -1$ (1p) $\rightarrow y = -1$ este asimptotă orizontală spre $-\infty$ (2p)

$\log(z) = -\infty$ (1p)
 $\log(z) = +\infty$ (1p) $\Rightarrow x = z$ este axiuplo
 verticală spre $\pm \infty$.
 (2p)

5.) f e continuă în $x=0 \Leftrightarrow l_s(0) = l_d(0) = f(0)$
 $l_d(0) = a(0)$ (2p)

$$l_{sf} = \frac{a}{b} \quad (4)$$

$$\ell(0) = \frac{a}{6} (2\pi) \Rightarrow \frac{a}{6} = \frac{a}{6} = \frac{2}{3} \quad (1p)$$

$$f(0) = \frac{2}{3} \left(\frac{1}{2} \right)$$

$$a = 4$$

$$b = 6$$

(2p)