Barem clasa 10 - profil real

Subiectul I (30p)

1.) $2x+1=\sqrt{(x+2)(3x+4)}$ (2p)

$4x^2+4x+1=3x^2+10x+8$ (2p)

$x^2-6x-7=0.$ (2p)

I $x_1=-1 \Rightarrow$ termenii progresiei sunt $1,-1,1$ (2p)

II $x_2=7 \Rightarrow$ termenii sunt $9,15,25$ (2p)

2.) $\frac{x_1}{x_2}=\frac{y_1}{y_2}$ (2p)

$\frac{m-2}{1}=\frac{2}{m-3}$ (2p)

$m^2-5m+4=0$ (2p)

$m_1=1$

$m_2=4.$ (4p)

3.) $4(3|x|-5)-2(5|x|+1)=2|x|+6-4(|x|-2)$ (2p)

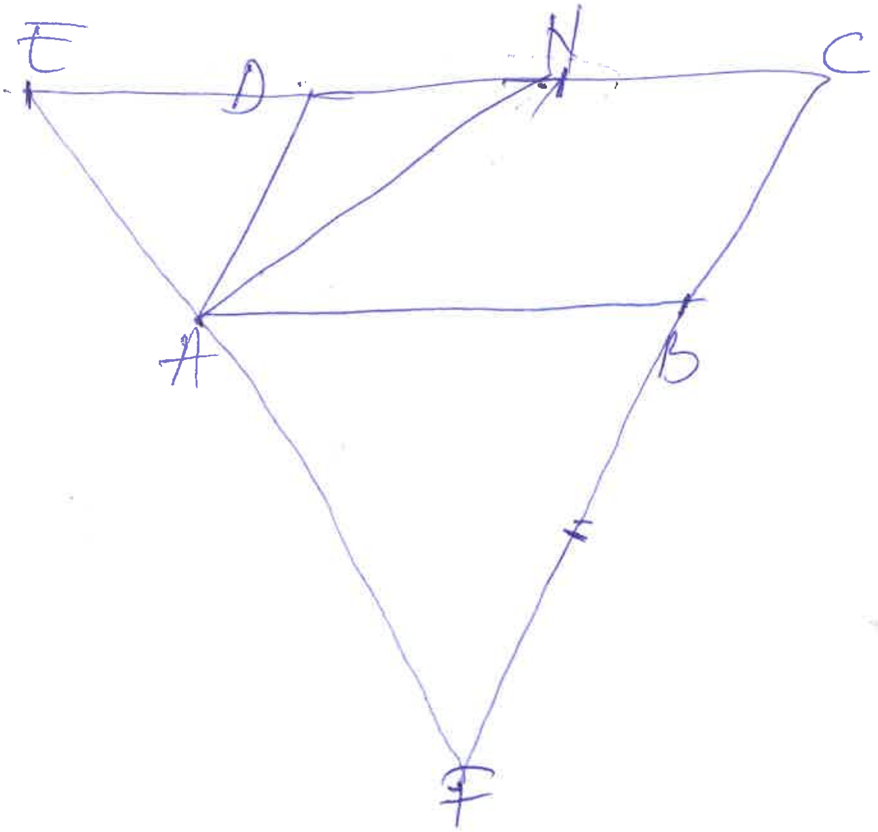
$2|x|-22=-2|x|+14$ (2p)

$4|x|=36$ (2p)

$|x|=9$ (2p)

$x=\pm 9.$ (2p)

Subiectul II (30p)



a.) În ΔEAD, $\vec{EA} = \vec{ED} + \vec{DA} = \frac{1}{2}\vec{DC} + \vec{CB} = \frac{1}{2}(\vec{DC} + 2\vec{CB})$ (3p)

În ΔABF, $\vec{AF} = \vec{AB} + \vec{BF} = \vec{DC} + 2\vec{CB}$ (3p)

$\Rightarrow \vec{AF} = 2\vec{EA}$ (2p)

$\Rightarrow \vec{AF}, \vec{EA}$ vectori coliniari $\Rightarrow E, A, F$ coliniare (2p)

b.) N mijloc DC $\Rightarrow$ D mijloc EN (2p)

În ΔAEN, AD mediană (1p)

$\Rightarrow \vec{AD} = \frac{1}{2}(\vec{AE} + \vec{AN})$ (2p)

$2\vec{AD} = \vec{AE} + \vec{AN} \Rightarrow 2\vec{AD} = -\frac{1}{2}\vec{AF} + \vec{AN}$ (3p)

$\vec{AN} = 2\vec{AD} + \frac{1}{2}\vec{AF}$ (2p)

2) Se observă o progresie aritmetică de rație $r=1$, $a_1 = 3x+\frac{1}{3}$, $a_n = 3x+\frac{37}{3}$) (2p)

$S_n = 121,(3) = \frac{364}{3}$ (1p)

$S_n = \frac{n(a_1+a_n)}{2}$ (2p)

$n = \frac{a_n - a_1}{r} + 1 = 13$ (2p)

$\frac{13\left(3x+\frac{1}{3}+3x+\frac{37}{3}\right)}{2} = \frac{364}{3} \quad / :13$ (2p)

$\frac{6x+\frac{38}{3}}{2} = \frac{28}{3}$ (2p)

$18x+38 = 56 \Rightarrow x = 1$ (1p)

Subiectul III (30p)

1) $(f\circ f\circ g)(x) = f(f(g(x))) = f(2(x+2)+1) =$

$= f(2x+5) = 2(2x+5)+1 = 4x+11$ (3p)

$(f\circ g\circ g)(x) = f(g(g(x))) = f(x+2+2) = f(x+4) =$

$= 2(x+4)+1 = 2x+9$ (3p)

$4x+11 = 2x+9$ (2p)

$x = -1$ (2p)

2) $\begin{cases} x_V = -\frac{b}{2a} = -\frac{2(m+1)}{2(m-1)} = -\frac{m+1}{m-1} & (1p) \\ y_V = -\frac{\Delta}{4a} = -\frac{12m+4}{4(m-1)} = -\frac{3m+1}{m-1} & (1p) \end{cases}$

$\Delta = 4(m+1)^2 - 4(m-1)m = 12m+4$ (1p)

$\begin{cases} x_V = -\frac{m-1+2}{m-1} = -1 - \frac{2}{m-1} \quad \Big| \cdot (-2) & (1p) \\ y_V = -\frac{3(m-1)+4}{m-1} = -3 - \frac{4}{m-1} & (1p) \end{cases}$

$\begin{cases} -2x_V = 2 + \frac{4}{m-1} & (1p) \\ y_V = -3 - \frac{4}{m-1} & (1p) \end{cases} \quad (+)$

$-2x_V + y_V = -1 \Rightarrow y_V = 2x_V - 1$ (1p)

$\Rightarrow$ V se află pe dreapta de ecuație $y = 2x - 1$ (2p)

3.) $\cos 2x = 2\cos^2 x - 1 \Rightarrow -\frac{1}{2} = 2\cos^2 x - 1$ (2p)

$2\cos^2 x = \frac{1}{2} \Rightarrow \cos^2 x = \frac{1}{4} \Rightarrow \cos x = \pm\frac{1}{2}$
dar $x \in \underline{C_{II}} \Rightarrow \cos x < 0$ $\Big| \Rightarrow \cos x = -\frac{1}{2}$ (3p)

$\cos 2x = 1 - 2\sin^2 x \Rightarrow -\frac{1}{2} = 1 - 2\sin^2 x$ (2p)

$2\sin^2 x = \frac{3}{2} \Rightarrow \sin^2 x = \frac{3}{4} \Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$
dar $x \in \underline{C_{II}} \Rightarrow \sin x > 0$ $\Big| \Rightarrow \sin x = \frac{\sqrt{3}}{2}$ (3p)