Barem clasa ~~10~~ 11 - profil real

Subiectul I (30p)

a.) $\sqrt[3]{6^{\log_9 81} - 9} =$ (2p)

$= \sqrt[3]{6^2 - 9} = \sqrt[3]{36-9} =$ (2p)

$= \sqrt[3]{27} = 3$ (1p)

b.) $\log_2\left(\sqrt{2}\cdot\sqrt{2+\sqrt{2}}\cdot\sqrt{2+\sqrt{2+\sqrt{2}}}\cdot\sqrt{2-\sqrt{2+\sqrt{2}}}\right) =$ (1p)

$= \log_2\left(\sqrt{2}\cdot\sqrt{2+\sqrt{2}}\cdot\sqrt{4-\left(\sqrt{2+\sqrt{2}}\right)^2}\right) =$ (1p)

$= \log_2\left(\sqrt{2}\cdot\sqrt{2+\sqrt{2}}\cdot\sqrt{2-\sqrt{2}}\right) =$ (1p)

$= \log_2\left(\sqrt{2}\cdot\sqrt{4-2}\right) =$ (1p)

$= \log_2\left(\sqrt{2}\cdot\sqrt{2}\right) = \log_2 2 = 1$ (1p)

~~Subiectul II~~

~~b)~~ 2) $(3+4i)z = 8-3i+5+i$ (2p)

$(3+4i)z = 13-2i$ (1p)

$z = \frac{13-2i}{3+4i}$ (2p)

$z = \frac{(13-2i)(3-4i)}{3^2-(4i)^2}$ (2p)

$$z=\frac{31-58i}{25}$$ (2p)

$$z=\frac{31}{25}-\frac{58}{25}i$$ (1p)

3) $1+\cos x=\pm 2$ (2p)

I $\cos x=1$ (1p)

$x=\pm\arccos 1+2k\pi$ (1p)

$x=2k\pi,\ k\in\mathbb{Z}$ (1p)

(1p)

II $\cos x=-3$

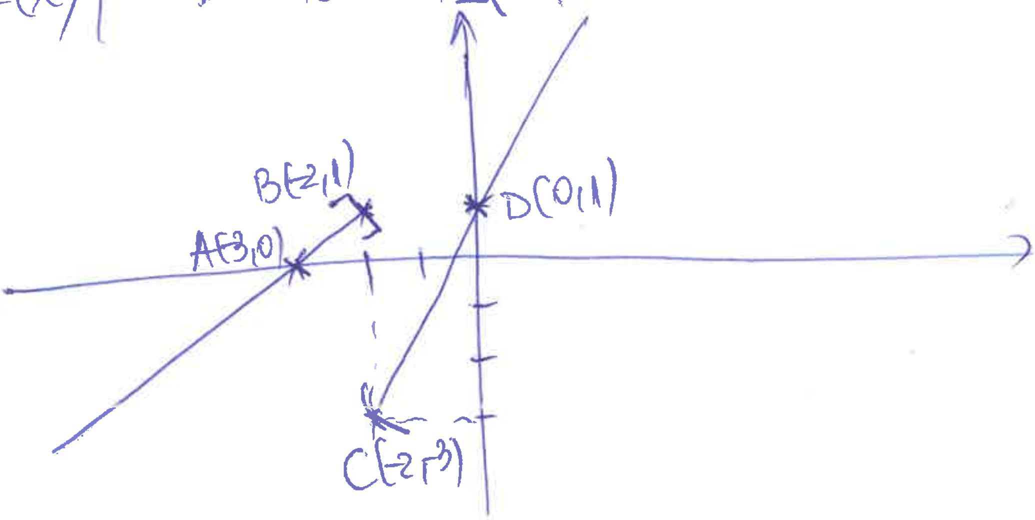
dar $-3\notin[-1,1]$ $\Rightarrow$ ec. nu are soluții (2p)

$S=\{2k\pi \mid k\in\mathbb{Z}\}$. (1p)

<u>Subiectul II</u> (30p)

Trasarea graficului funcției (3p)

x	$-\infty$	-3	-2	0	$+\infty$
$f(x)$	$\nearrow$	0 $\nearrow$	1] [-3 $\nearrow$	1	$\nearrow$



(3p)

f nu este injectivă (2p)

f este surjectivă (2p)

2.) $C_n^0 + C_n^2 + C_n^4 + \ldots = 2^{n-1} = 128$ (2p)

$2^{n-1} = 2^7 \Rightarrow n = 8$ (1p)

$T_{k+1} = C_n^k \cdot a^{n-k} \cdot b^k =$ (1p)

$= C_8^k \cdot \left(x\sqrt[4]{x}\right)^{8-k} \cdot \left(x^{-\frac{1}{2}}\right)^k =$ (1p)

$= C_8^k \cdot x^{\frac{5}{4}(8-k) - \frac{k}{2}}$ (1p)

$= C_8^k \cdot x^{\frac{40-7k}{4}}$ (1p)

$x^{\frac{40-7k}{4}} = x^3 \Rightarrow \frac{40-7k}{4} = 3$ (2p)

$k = 4$ (1p)

$T_5 = C_8^5 \cdot x^3$

3.) $\begin{cases} S = x_1 + x_2 = -\frac{b}{a} = 1 \\ P = x_1 x_2 = \frac{c}{a} = 1 \end{cases}$ (2p)

$x_1^2+x_2^2=S^2-2P=-1$ (2p)

$x_1^4+x_2^4=(x_1^2+x_2^2)^2-2x_1^2x_2^2=$ (2p)

$=(-1)^2-2\cdot 1=-1$

x_1 soluție a ec $\Rightarrow x_1^2-x_1+1=0$ (1p)

$x_1^2=x_1-1$

$x_1^3=x_1^2-x_1=x_1-1-x_1=-1.$ (1p)

$\Rightarrow x_2^3=-1.$

$x_1^{2019}+x_2^{2019}=(x_1^3)^{673}+(x_2^3)^{673}=$ (2p)

$=(-1)^{673}+(-1)^{673}=$

$=-2.$

Subiectul III (30p)

a) $\sqrt{3-x}=\sqrt{10-x}-1 \Rightarrow \sqrt{3-x}+1=\sqrt{10-x}$ (2p)

CE: $\begin{cases} 3-x\geq 0 \\ 10-x\geq 0 \end{cases} \Rightarrow x\in \cancel{[3,10]}\ (-\infty,3]$ (2p)

(2p)

$3-x+2\sqrt{3-x}+1=10-x$

$2\sqrt{3-x}=6 \quad |:2$ (2p)

$\sqrt{3-x}=3$ (2p)

$3-x=9 \Rightarrow x=-6\in(-\infty,3] \quad S=\{-6\}$

b) $4^{x-\sqrt{x^2-5}} - 12 \cdot \frac{2^{x-\sqrt{x^2-5}}}{2} + 8 = 0.$

Notăm $2^{x-\sqrt{x^2-5}} = t$, $t \geq 0$, $x^2-5 \geq 0$.

$t^2 - 6t + 8 = 0$

$t_1 = 2$

$t_2 = 4$

I $2^{x-\sqrt{x^2-5}} = 2 \Rightarrow x - \sqrt{x^2-5} = 1$

$x - 1 = \sqrt{x^2-5}$

$x^2 - 2x + 1 = x^2 - 5$

$x = 3$ care convine

II $2^{x-\sqrt{x^2-5}} = 4 \Rightarrow x - \sqrt{x^2-5} = 2$

$x - 2 = \sqrt{x^2-5}$

$x^2 - 4x + 4 = x^2 - 5$.

$x = \frac{9}{4}$ care convine

c.) $2\lg(3x-4) + 2\lg(4x-8) = 4 \quad |:2$

$\lg(3x-4) + \lg(4x-8) = 2$ (1p)

CE: $\begin{cases} 3x-4>0 \\ 4x-8>0 \end{cases} \Rightarrow x \in (2, \infty)$ (2p)

$\lg(3x-4)(4x-8) = \lg 100$ (2p)

$$12x^2-40x+32=100 \qquad (2p)$$

$$12x^2-40x-68=0 \quad |:4$$

$$3x^2-10x-17=0 \qquad (2p)$$

$$\Delta=100+12\cdot 17=304.$$

$$x_{1,2}=\frac{10\pm\sqrt{304}}{6}=\frac{10\pm 4\sqrt{19}}{6}=\frac{5\pm 2\sqrt{19}}{3}$$

$$x_1=\frac{5-2\sqrt{19}}{3}\notin(2,\infty)$$

$$x_2=\frac{5+2\sqrt{19}}{3}\in(2,\infty) \quad (1p)$$

$$S=\left\{\frac{5+2\sqrt{19}}{3}\right\}.$$