Barem clasa a 12-a profil real

Subiectul I (30p)

a.) $\begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix} = 0$ (1p)

$\begin{vmatrix} 4 & 2 & 1 \\ m & 2 & 1 \\ -2 & 1-m & 1 \end{vmatrix} = 0$ (1p)

(3p)

$\Rightarrow 8 + m - m^2 - 4 + 4 - 4 + 4m - 2m = 0.$

(2p)

$-m^2 + 3m + 4 = 0$

(2p)

$m_1 = -1$

$m_2 = 4$

$m \in \{-1, 4\}$ (1p)

b.) $A_{\Delta ABC} = \frac{1}{2}|\Delta| \Rightarrow$ (1p)

$|\Delta| = 36 \Rightarrow \Delta = \pm 36.$ (2p)

De la a), $\Delta = -m^2 + 3m + 4.$

I. $-m^2 + 3m + 4 = +36$ (1p)

$m^2 - 3m + 32 = 0$

$\Delta < 0 \Rightarrow$ ec. nu are soluții reale (2p)

1 $-m^2+3m+4=-36$ (1p)

$m^2-3m-40=0.$ (1p)

$m_1=8$

$m_2=-5$ (2p)

2.) $\Delta \xlongequal{L_1+L_2+L_3} \begin{vmatrix} 3x+9 & 3x+9 & 3x+9 \\ x & x+1 & 2x+5 \\ 2x+8 & 2x+6 & x+1 \end{vmatrix} =$ (2p)

$\xlongequal[C_3-C_1]{C_2-C_1} (3x+9) \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & x+5 \\ 2x+8 & -2 & -x-7 \end{vmatrix} =$ (3p)

$=(3x+9)\cdot 1\cdot(-1)^{1+1}\cdot \begin{vmatrix} 1 & x+5 \\ -2 & -x-7 \end{vmatrix} =$ (2p)

$=(3x+9)(x+3)$ (1p)

$\Delta=0 \Rightarrow x_1=-3$

$x_2=-3$ (2p)

Subiectul II (30p)

$A(-1)=\begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$ (2p)

a) $\det A(-1)=\begin{vmatrix} 2 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix}=$ (2p)

$=2-1+1-1-2+1=$ (3p)

$=0$ (3p)

b.) $\begin{cases} \cancel{\det A(a)}=0 \\ \Delta_c\neq 0 \end{cases}$ (2p)

$\det A(a)=\begin{vmatrix} 2 & -1 & 1 \\ 1 & 1 & a \\ 1 & -1 & 1 \end{vmatrix}=a+1.$ (2p)

$a+1=0 \Rightarrow a=-1$ (1p)

$\Delta p=\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}\neq 0$ (2p)

$\Delta c=\begin{vmatrix} 2 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & b \end{vmatrix}=3b-3$ (2p)

$b\neq 1 \Rightarrow b\in\mathbb{R}-\{1\}$ (1p) c.) $\begin{cases} \det A(a)=0 \\ \Delta_c = 0 \end{cases}$ (1p)

De la b) $\Rightarrow a=-1$ (2p)

$b=1$

x, y nec principale

~~z =~~ α nec secundara (1p)

ec 1,2 principale

$\begin{cases} 2x-y=1-\alpha \\ x+y=-1+\alpha \end{cases}$

$x=0$

$y=\alpha-1$

$S=\{(0, \alpha-1, \alpha)\}, \alpha\in\mathbb{R}$ (2p)

$E=-1+\alpha+\alpha^2$ este minimă dacă (1p)

$\alpha=-\frac{b}{2a}=-\frac{1}{2}$. (2p)

$S=\{(0, -\frac{3}{2}, -\frac{1}{2})\}$. (1p)

Subiectul III

1.) a) $\lim\limits_{x\to 1}\frac{f(x)-f(1)}{x-1}=f'(1)$ (2p)

$f'(x)=2\ln x+2-2x$ (2p)

$f'(1)=0$ (1p)

b.) $f'(x) = 2\ln x + 2 - 2x$ (2p)

$f''(x) = \frac{2}{x} - 2 = \frac{2-2x}{x}$ (3p)

c.) $f''(x) = 0 \Rightarrow x = 1$. (1p)

x	0		1		$+\infty$
$f''(x)$	$+$	$+\ +$	0	$-\ -$	$-$
$f'(x)$		$\nearrow$	0	$\searrow$	

(2p)

$f'(x)$ este crescătoare pe $(0,1]$

$f'(x)$ este descrescătoare pe $[1,\infty)$ (2p)

$\Rightarrow x=1$ punct de maxim pt $f'(x)$ (2p)

$f'(x) \le f'(1)$, $(\forall)\ x>0$ (2p)

$f'(x) \le 0$. (1p)

d.)

x	0		1		$+\infty$
$f'(x)$	$-$	$-\ -$	0	$-\ -$	$-$
$f(x)$		$\searrow$	2	$\searrow$	

(3p)

f e strict descrescătoare pe $(1,\infty)$ (1p)

$\Rightarrow f(x) < f(1)$ (2p)

$f(x) < 2$, $(\forall)\ x>1$ (1p)

$2x\ln x - x^2 + 3 < 2$

$2x\ln x < x^2 - 1 \ /: x>0 \Rightarrow 2\ln x < x - \frac{1}{x}$, $(\forall)\ x>1$ (3p)